

$$\textcolor{red}{d}_1(P, F_1) + \textcolor{red}{d}_2(P, F_2) = 2a$$

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

$$\sqrt{(x-c)^2 + y^2} = 2a - \sqrt{(x+c)^2 + y^2}$$

$$\left[\sqrt{(x-c)^2 + y^2} \right]^2 = \left[2a - \sqrt{(x+c)^2 + y^2} \right]^2$$

$$(x-c)^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + (x+c)^2 + y^2$$

$$x^2 - 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2 + y^2} + x^2 + 2cx + c^2 + y^2$$

$$-4cx = 4a^2 - 4a\sqrt{(x+c)^2 + y^2}$$

$$4a\sqrt{(x+c)^2 + y^2} = 4a^2 + 4cx$$

$$a\sqrt{(x+c)^2 + y^2} = a^2 + cx$$

$$\left[a\sqrt{(x+c)^2 + y^2} \right]^2 = (a^2 + cx)^2$$

$$a^2 \left[(x+c)^2 + y^2 \right] = a^4 + 2a^2cx + c^2x^2$$

$$a^2x^2 + 2a^2cx + a^2c^2 + a^2y^2 = a^4 + 2a^2cx + c^2x^2$$

$$a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$$

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

$$\frac{(a^2 - c^2)x^2}{a^2(a^2 - c^2)} + \frac{a^2y^2}{a^2(a^2 - c^2)} = \frac{a^2(a^2 - c^2)}{a^2(a^2 - c^2)}$$

$$\frac{x^2}{a^2} + \frac{y^2}{(a^2 - c^2)} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

